Scaling laws of strategic behavior and size heterogeneity in agent dynamics

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We consider the financial market as a model system and study empirically how agents strategically adjust the properties of large orders in order to meet their preference and minimize their impact. We quantify this strategic behavior by detecting scaling relations between the variables characterizing the trading activity of different institutions. We also observe power-law distributions in the investment time horizon, in the number of transactions needed to execute a large order, and in the traded value exchanged by large institutions, and we show that heterogeneity of agents is a key ingredient for the emergence of some aggregate properties characterizing this complex system.

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I. INTRODUCTION

Scaling [1] is a key concept in the modeling of complex systems, and it is found in a wide range of systems and phenomena ranging from supercooled liquids [2] to heartbeat dynamics [3] and to the metabolism of organisms [4]. Scaling has also been observed in social and economic systems such as, for example, the price impact of a single transaction occurring in a financial market [5] and, more recently, the growth of cities [6]. Power-law distributions are also pervasive in complex systems investigation and modeling [7]. Power laws are widely observed in physics, biology, computer science, demography, earth sciences, economics and finance, psychology, and the social sciences. In this paper, we describe observables quantitatively characterizing the strategic behavior of heterogeneous agents in terms of scaling relations and power-law distributions.

The dynamics of many socioeconomic systems is determined by the decision-making process of agents. The decision process depends on the agents' characteristics, such as preferences, risk aversion, behavioral biases, etc [8,9]. In addition, in some systems the size of agents can be highly heterogeneous, leading to very different impacts of agents on the system dynamics [10–15]. The large size of some agents poses challenging problems to agents who want to control their impact, either by forcing the system in a given direction or by hiding their intentionality. It is likely that large agents impact the system in a way that is significantly different from small ones. Indeed, small agents can easily hide their intentionality, while for large agents this is not so easy and they must adopt strategies taking into account their own effect because revealing their intention could decrease their fitness. Financial markets are an ideal system to investigate this problem. There is empirical evidence that financial market participants are very heterogeneous in size. For example, the sizes of banks [14] and mutual funds [15] follow Zipf's law, i.e., the probability that the size of a participant is larger than x decays as $1/x$ [11]. As a consequence, large investors usually need to trade large quantities that can significantly affect prices. The associated cost is called market impact [5,16–19]. For this reason, large investors refrain from revealing their demand or supply and they typically trade their large orders incrementally over an extended period of time. These large orders are called *packages* [23,24] or *hidden orders* and are split into smaller trades as the result of a complex optimization procedure which takes into account the investor's preference, risk aversion, investment horizon, etc.

The empirical characterization of the statistical properties of packages is a difficult task due to the difficulty of accessing proprietary data. A few studies have been performed by using limited data sets of packages exchanged by a few financial institutions. Here we tackle the problem from a different perspective. By making use of a special database in which the identity of the buyer and the seller of each transaction is disclosed (at least in some form; see below), we develop a statistical algorithm able to identify in a statistical way the presence of packages in the activity of a market participant. While some uncertainty is unavoidably present due to the statistical nature of our algorithm, our investigation is at the level of the whole market because essentially all the trades are investigated. Specifically, here we investigate the trading activity of a large fraction of the financial firms exchanging a financial asset at the Spanish stock market [Bolsas y Mercados Españoles (BME)] in the period 2001–2004. We aim for a comprehensive approach analyzing the overall dynamics of all packages exchanged in the market. After the identification of the packages, we study the statistical properties of these packages in terms of distributional properties and scaling laws. We also investigate the role of participants' heterogeneity in explaining the observed statistical regularities.

In Sec. II we introduce our database and the algorithm we developed to identify in a statistical way the packages exchanged by each firm in the market. In Sec. III, we investigate the statistical properties of the variables characterizing the packages, and in Sec. IV we study the scaling relations between the variables of the packages. In Sec. V we consider the problem of whether the statistical regularities detected in

Secs. III and IV hold for each firm individually or if they are an effect of the heterogeneity of firms. Finally Sec. VI gives the conclusions.

II. DATA AND DETECTION METHOD

Our database of the electronic open market Sistema de Interconexión Bursátil Electrónico (SIBE) allows us to follow each transaction performed by all the firms registered at the BME. In 2004 the BME was the eighth in the world in market capitalization. We consider firm transactions only on the stocks Banco Bilbao Vizcaya Argentaria (BBVA), Banco Santander Central Hispano (SAN), and Telefónica (TEF) which are three highly liquid stocks. The investigated period is 2001–2004. We do not consider other stocks because we have verified that the number of detected firm packages is too small for a careful statistical estimation. In this market, firms are local and foreign credit entities and investment firms which are members of the stock exchange and they are the only firms entitled to trade. Orders to buy and sell are entered into the market only through members of the stock market. Approximately 75% of them are major financial institutions and 25% are established securities dealers. Both types may trade on their own behalf and also on behalf of other individuals and/or institutions that are not members of the market. It is important to stress that firms are not necessarily quoted companies (stocks) but rather are the only institutions entitled to trade stocks directly. The interest of this work is in part related to the availability of data on the firms' activity rather than on stock macroscopic variables (price, volume, etc.). In this paper we consider only the most active firms defined by the criterion that each firm made at least 1000 trades per year and was active at least 200 days per year. The number of firms is 50 (BBVA), 55 (SAN), and 61 (TEF). These firms are involved in 81–86 % of the transactions. The series under study is the series of signed traded value. For each firm and for each stock we construct a series composed of all the trades performed by the firm with a value $+v$ for a buy trade and $-v$ for a sell trade, where v is the value (in euros) of the traded shares.[1]

Our database does not contain direct information on packages, so that this information must be statistically inferred from the available data. Since we do not have information on clients but only on firms, we develop a detection algorithm which is not sensitive to small fluctuations in the buy-sell activity of a firm. The algorithm is adapted from Ref. [25], where it was introduced to study patchiness in the non-stationary dynamics of the human heart rate, and it detects time segments in the inventory time evolution of a firm when the firm acts as a net buyer or seller at an approximately constant rate. The algorithm works as follows. One moves a sliding pointer along the signal and computes the mean of the subset of the signal to the left and to the right of the pointer. From these mean values one computes a t statistic and finds the position of the pointer for which the t statistic is maximal. The significance level of this value of t is defined as the probability of obtaining it or a smaller value in a random sequence. One then chooses a threshold (in our case 99%) and the sequence is cut if the significance level is larger than the threshold. The cut position is the boundary between two consecutive patches. The procedure continues recursively on the left and right subsets created by each cut. Before a new cut is accepted, one also computes t between the right-hand new segment and its right neighbor and t between the left-hand new segment and its left neighbor, and one checks if both values of t are statistically significant according to the selected threshold. The process stops when it is not possible to make a new cut with the selected significance.

We call the detected segments *patches*. Since firms act simultaneously as brokers for many clients, it is rather frequent that in a patch not all the transactions have the same sign. However, a vast majority of firm inventory time series can be partitioned into patches with a well-defined direction toward buying or selling. This is probably due to the fact that in most cases the trading activity of a firm is dominated by the activity of one big client.

In the present study, we are mainly interested in *directional patches*, i.e., patches where the trader consistently buys or sells a large amount of shares. Our working hypothesis is that each of these patches contains at least one package. To clarify this point, consider the case of a firm submitting two packages with the same sign and the same trading velocity. Our detection algorithm will detect only one long package. We wish to exclude patches in which the inventory of the firm is diffusing randomly, without a drift. To this end, for each patch we compute the total value purchased, V_b, the total value sold, V_s, and the total value $V=V_b+V_s$. We then consider a patch as directional when either $V_b/V>\theta$ (buy patch) or $V_s/V>\theta$ (sell patch). The parameter θ can be varied and in the present study we set it to $\theta=75\%$. We obtain similar results for different values of θ such as 85% and 95%. Finally, in the present paper we consider patches with at least ten trades. An example of an inventory time series and the output of the segmentation algorithm is shown in Fig. 1.

The characterizing variables of a directional patch are the time length T (in seconds) of the patch, measured as the time interval between the first and the last orders of the patch, the traded value V_m, and the number N_m of trades characterizing the patch. For example, N_m is the number of buy trades and V_m is the purchased value for buy patches.

III. DISTRIBUTIONAL PROPERTIES OF PATCHES

We investigate first the distributional properties of the patches identified by our algorithm. Figure 2 shows the distributions of T, N_m, and V_m for the three investigated stocks. The asymptotic behavior of all three distributions can be approximated by a power-law function $P(X)\sim 1/X^{\zeta_X+1}$, where X can be T, N_m, or V_m, and ζ_X is the exponent characterizing the power-law behavior. A summary of the esti-

[1]We have repeated the analysis by considering the time series of volume, i.e. number of shares, rather than of value. The results are essentially the same and the exponents shown in Table I for the value time series are statistically indistinguishable from the ones obtained for the volume time series.

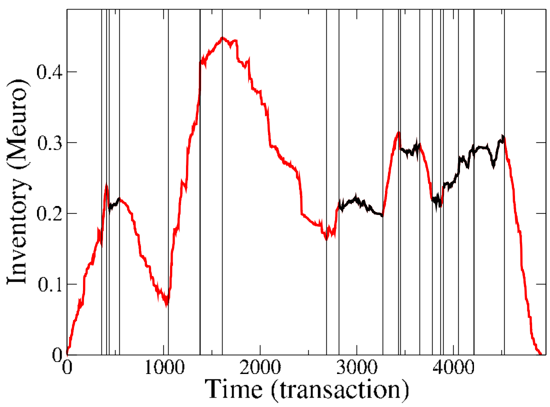

FIG. 1. (Color online) Example of an inventory time series. The series refers to a particular firm trading Santander. The vertical lines indicate the position where our algorithm predicts the boundary between two patches. Directional patches are shown as red lines. Due to their statistical nature, in each patch there are buy (with a total traded value V_b) and sell (with a total traded value V_s) trades. We consider directional patches, i.e., patches where either $V_b/V > \theta$ (buy patch) or $V_s/V > \theta$ (sell patch), where $V=V_b+V_s$. For buy patches $V_m=V_b$ whereas for sell patches $V_m=V_s$. In the present study we set θ=75%. The black patches are not directional and are not considered in the rest of the paper.

mated exponents is shown in Table I from which one can conclude that $\zeta_{V_m} \simeq 2$, $\zeta_{N_m} \simeq 1.8$, and $\zeta_T \simeq 1.3$. Our analysis makes explicit the presence of a very broad distribution for the three variables characterizing a patch. In fact the very low value of the exponents is consistent with the conclusion that T and N_m belong to the domain of Lévy-stable distributions. This result indicates that in the market there is a huge heterogeneity in the scales characterizing the trading profiles of the investors. The volume of the packages is likely to be related to the size of the investor. Large investors need to trade large packages to rebalance their portfolio. Gabaix *et al.* [26] developed a theory which predicts that package size should be power-law distributed with an exponent $\zeta_{V_m}=3/2$. The value we find for $\zeta_{V_m} \simeq 2$ is slightly larger than the one predicted by them. On the contrary, the value $\zeta_{N_m}=3$ derived by the theory in [26] is significantly larger than our estimate ($\zeta_{N_m} \simeq 1.8$). Finally, the power-law distribution of packages time length T might reflect the heterogeneity of time scales among investors. The power-law distribution for T has been recently suggested in stylized models of investment decisions [20–22] and is quantitatively compatible with the ones obtained by using specialized databases describing the investment packages of large investors [23,24] (see Fig. 2). The Gabaix *et al.* theory [26] predicts the value $\zeta_T=3$, which is significantly larger than our value ($\zeta_T \simeq 1.3$). The role of size heterogeneity in the emergence of power-law distributions will be considered below.

IV. SCALING RELATIONS

To complete our characterization of firm patches, we now consider the relation between the variables characterizing

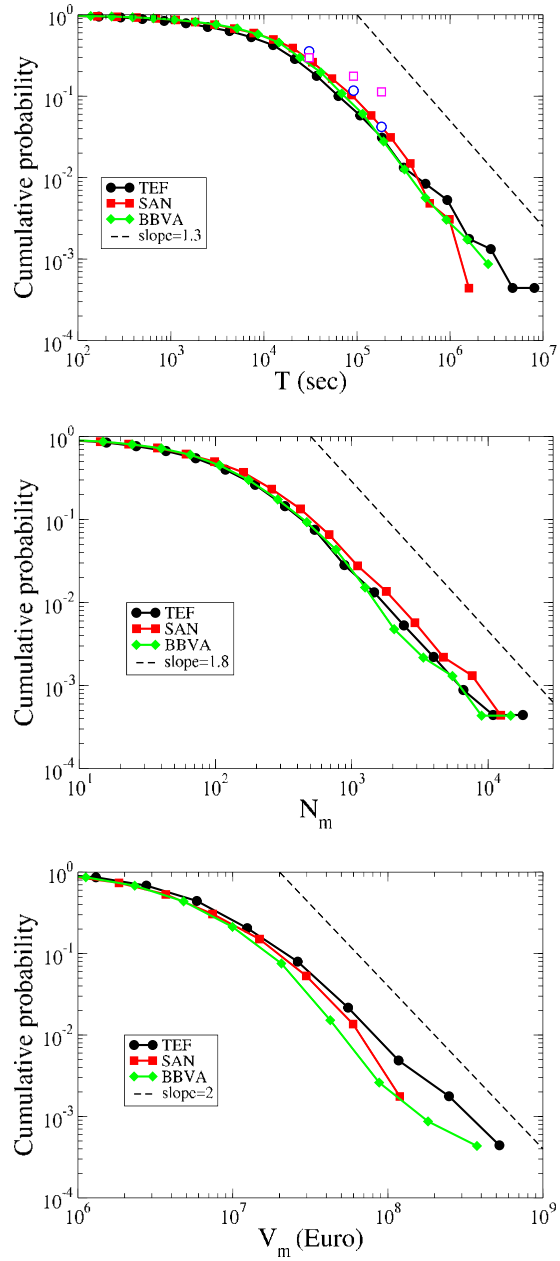

FIG. 2. (Color online) Distribution of T, N_m, and V_m for the stocks Banco Bilbao Vizcaya Argentaria (BBVA), Banco Santander Central Hispano (SAN), and Telefónica (TEF). In the panel showing the distribution of T we plot the distribution of packages reported in the literature on packages. Specifically, empty blue circles are results from Ref. [23] for packages traded at the New York Stock Exchange and empty magenta squares are results from Ref. [24] for packages traded at the Australian Stock Exchange.

each patch. Specifically, by applying the principal component analysis (PCA) to the set of points with coordinates $(\log T, \log N_m, \log V_m)$, we investigate the scaling or allometric relations between any two of the above variables, i.e.,

$$N_m \sim V_m^{g_1}, \quad T \sim V_m^{g_2}, \quad N_m \sim T^{g_3}. \tag{1}$$

Figure 3 shows the scatter plots and the contour plots for the stock Telefónica. In all three cases a clear dependence be-

TABLE I. Summary of the properties of detected patches. The number in parentheses in the column headings is the number of patches detected for the considered stock. Rows 1–3: Tail exponents of the distribution of T, N_m, and V_m estimated with the Hill estimator (or maximum likelihood estimator). In parentheses we report the 95% confidence interval. Rows 4–6: Exponents of the scaling relations defined in Eq. (1). The exponents are estimated with PCA and the errors are estimated with a bootstrap algorithm. In parentheses we report the 95% confidence interval. Rows 7–9: Percentage of firms with at least ten patches for which one cannot reject the hypothesis of log-normality with 95% confidence according to a Jarque-Bera test. The numbers in parentheses are the numbers of firms for which one cannot reject the hypothesis of log-normality divided by the number of firms used in the test. Rows 10 and 11: Percentage of firms with at least ten patches for which the 3D PCA gives the first (λ_1) or second (λ_2) eigenvalue larger than the corresponding first or second eigenvalue obtained from the PCA of the pool of all the patches. The numbers in parentheses are the absolute numbers of firms.

	BBVA (2104)	SAN (2086)	TEF (2062)
ζ_{V_m}	2.3 (1.9;2.7)	2.0 (1.7;2.3)	1.9 (1.6;2.2)
ζ_{N_m}	2.0 (1.7;2.3)	1.7 (1.4;2.0)	1.7 (1.4;2.0)
ζ_T	1.5 (1.3;1.7)	1.5 (1.3;1.7)	1.2 (1.0;1.4)
g_1	1.08 (1.05;1.12)	1.06 (1.01;1.10)	1.07 (1.04;1.11)
g_2	1.81 (1.69;1.93)	1.81 (1.68;1.94)	2.00 (1.88;2.14)
g_3	0.68 (0.65;0.71)	0.68 (0.65;0.70)	0.62 (0.59;0.64)
T	75 (15/20)	63 (17/27)	77 (24/31)
N_m	90 (18/20)	100 (27/27)	100 (31/31)
V_m	90 (18/20)	100 (27/27)	94 (29/31)
λ_1	90 (18/20)	85 (23/27)	87 (27/31)
λ_2	15 (3/20)	18 (5/27)	22 (7/31)

tween the variables is seen. PCA analysis shows that the first eigenvalue explains on average 91%, 83%, and 89% of the variance for the first, second, and third scaling relations, respectively, indicating a strong correlation between the variables. The estimated exponents (see Table I) are consistent for different stocks so that the scaling relations are

$$N_m \sim V_m^{1.1}, \quad T \sim V_m^{1.9}, \quad N_m \sim T^{0.66}. \tag{2}$$

The presence of scaling relations between the variables was first suggested in Ref. [26], but it is worth noting that the theory developed in that paper predicts $g_1 = g_2 = 1/2$ and $g_3 = 1$, and these values are quite different from the ones we estimate from data. The first scaling relation in (2) indicates that the number of transactions into which a package is split is approximately proportional to the total traded value of the package. This implies that the mean transaction volume is roughly independent of the size of the package. We have independently verified that the mean transaction size is roughly independent of V_m (see Fig. 4). This mean value is on average determined by the size of the available volume at the best quote, indicating that the trader does not trade orders larger than the volume available at the best quote, probably to avoid being too aggressive. In fact, it has been recently shown that the orders initiating transactions are almost always smaller than or equal to the size available at the best price [27]. Thus, even if a trader has a large hidden order to trade, the size of each transaction will be determined by the best available volume, i.e., the liquidity of the market.

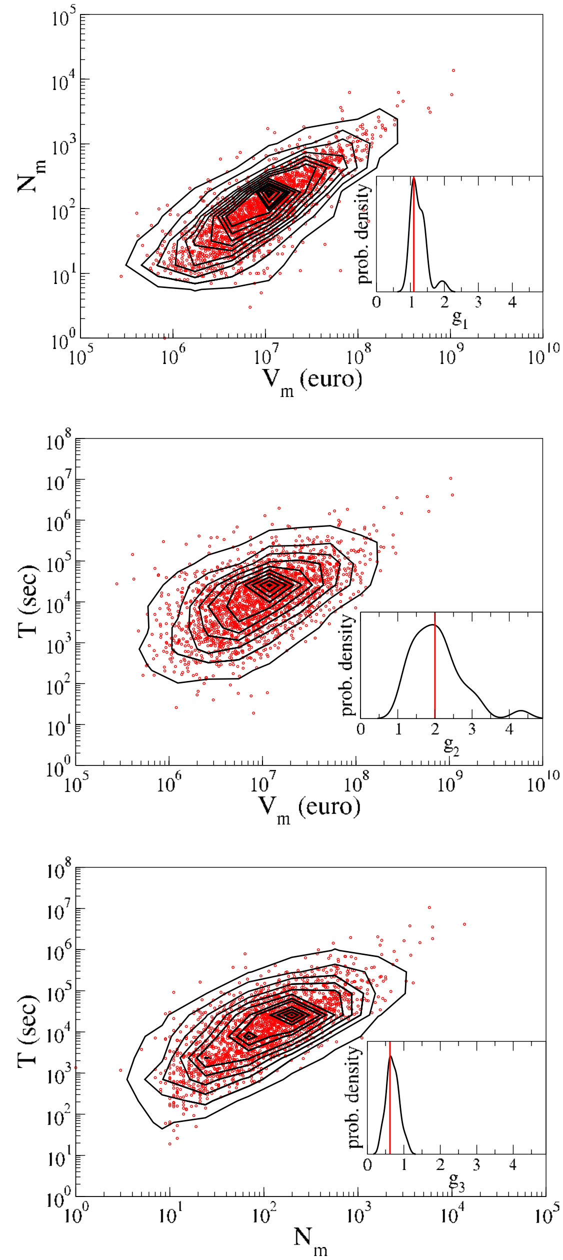

FIG. 3. (Color online) Scatter plots of the variables T, N_m, and V_m for Telefónica. The black lines are contour lines of the bivariate probability density function. The insets show the probability density functions of the three exponents g_1, g_2, and g_3 describing the scaling relations of Eq. (1) computed on the patches of individual firms with at least ten patches. The red vertical lines indicate the values of the scaling exponents computed in the pool of all firms and reported in rows 4–6 of Table I. It is worth noting that the dispersion of g_2 is significantly larger than that for the other two exponents.

We consider the relation between the three variables together by performing a PCA on the set of points describing the patches and identified by the coordinates

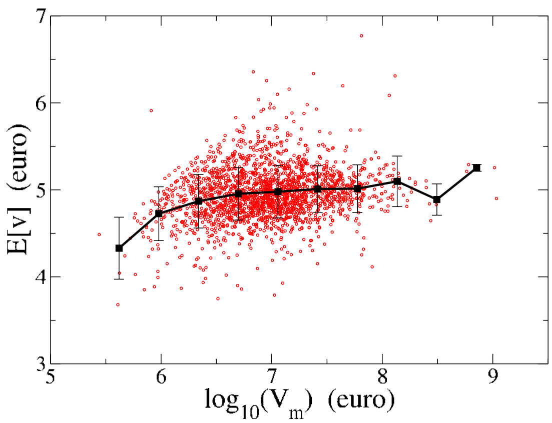

FIG. 4. (Color online) Scatter plot of the mean transaction volume in a patch $E[v]$ as a function of the total volume V_m of the patch. The considered stock is Telefónica. The black line shows the mean value of $E[v]$ for points in equal-sized bins of V_m. Error bars are one standard deviation.

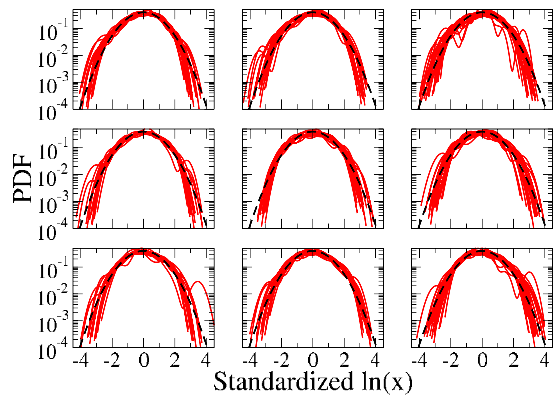

FIG. 5. (Color online) Probability density function of the standardized logarithm of the variables T, N_m, and V_m of the firms for which the Jarque-Bera test of log-normality cannot be rejected. Specifically, for each stock and each variable we consider the firms for which the log-normal hypothesis cannot be rejected (see Table I and text). For each of these firms we compute the logarithm of the variable; we subtract the mean value and divide by the standard deviation. According to the null hypothesis these normalized variables should be Gaussian distributed. In the figure we plot on a semilogarithmic scale the probability density functions for each firm (continuous lines) and we compare them with the Gaussian probability density function (dashed line). Each column refers to a stock (from left to right, BBVA, SAN, TEF) and each row refers to a variable (from top to bottom T, N_m, and V_m).

$(\log T, \log N_m, \log V_m)$ [28]. The set of points effectively lies on a two-dimensional manifold which has one dimension much larger than the other. The fact that the first eigenvalue is large indicates that one factor dominates the trading strategy. The scaling relations of the three variables associated with the first eigenvalue of the PCA provide an estimation of the exponents ($g_1 \simeq 1.2$, $g_2 \simeq 1.8$, and $g_3 \simeq 0.67$ for Telefónica) which, unlike in the bivariate case, are of course coherent among themselves and only slightly different from the ones obtained from the bivariate analysis.

V. THE ROLE OF FIRM HETEROGENEITY

We now go back to the problem of assessing the role of firm heterogeneity. The first scientific question is as follows: Is the fat-tailed distribution of T, N_m, and V_m due to the fact that individual firms place heterogeneously sized packages, or is this an effect of the aggregation of many different firms together? To answer this question we test the hypothesis that the patches identified for a given firm trading a given stock are log-normally distributed. The test (see Table I) shows that for most of the trading firms we cannot reject the hypothesis that the patches have characteristic sizes distributed log-normally. Since we reject the log-normal hypothesis for the pool obtained by considering all the firms, we conclude that the power-law distribution of T, N_m, and V_m is due to a heterogeneity in patch scale *between* different firms rather that *within* each firm. In Fig. 5 we show the probability density functions of T, N_m, and V_m for those firms for which the log-normal hypothesis cannot be rejected. The figure qualitatively confirms the result of the statistical test.

The second scientific question concerns the role of firm heterogeneity for scaling laws. To assess the role of heterogeneity, for each firm we compute the exponents g_1, g_2, and g_3 of the bivariate relations of Eq. (1) (see insets of Fig. 3). We observe that the exponents obtained for each firm are distributed around the corresponding value of the exponent obtained for the pool. This result indicates that the bivariate scaling relations are not an effect of the aggregation but are observed, on average, also for individual firms.

Finally we study how the heterogeneity affects the three-dimensional (3D) allometric relations. To this end we perform the 3D PCA on the patches detected for each firm with at least ten patches. We find that for the vast majority of firms (see rows 10 and 11 of Table I) the first eigenvalue is larger than the first eigenvalue of the 3D PCA of the pool of all the firms. Moreover, the second eigenvalue obtained from PCA of individual firms is very often smaller than the second eigenvalue of the pool sample. Our analysis suggests that patches of an individual firm are essentially explained by one eigenvector, i.e., one size variable is enough to explain the other two. Our investigation also suggests that the second eigenvalue of the pool, which explains 15% of the variance, is mainly due to the heterogeneity between firms. In other words, imagine representing each patch as a point in a tridimensional space with coordinates $\log T$, $\log V_m$, and $\log N_m$. The 3D PCA on the pool of the patches of all the firms indicates that these points form an approximately bidimensional cloud having one dimension much larger than the other. If one now considers only the points representing the patches of a given firm, one observes that these points lie approximately on a straight line with a direction close to the direction of elongation of the cloud. The bidimensional cloud representing the pool of patches can be seen as the aggregation of many straight lines (one for each firm).

VI. CONCLUSIONS

In conclusion, our comprehensive investigation of packages traded at the BME shows the presence of statistical laws describing the properties of the trading activity of market participants. The empirical evidence that the variables describing the size of the packages are power-law distributed with a low exponent indicates that many different time scales are present in the market. In other words, the time horizon of the investment strategy ranges from a few minutes to many months. This multiscale property of the market dynamics has often been suggested, but, to the best of our knowledge, our study is the first empirical evidence that different scales are present at the level of investment strategies. This evidence has rarely been taken into consideration in agent-based models of financial markets. The scaling laws between the variables characterizing the packages are a starting point for understanding the optimization process traders use to minimize their own impact. Finally, our investigation is useful in understanding the role of firms' heterogeneity in the statistical laws of the packages. We have shown that heterogeneity of firms has an essential role for the emergence of power-law tails in the investment time horizon, in the number of transactions, and in the traded value exchanged by packages. This suggests that the multiscale property mentioned above is the result of a heterogeneity of scales between different market participants. In contrast, scaling laws between the variables characterizing each package are essentially the same across different firms, with the possible exception of the relation between T and V_m, perhaps reflecting different degrees of aggressiveness of firms. This suggests that market participants are roughly homogeneous in the optimization process they follow to minimize their impact.

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